Science and Math in the Orchestra Classroom

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D'Addario

Why Science and Math?

- Motivation for students to view music more globally
- Cross-curricular teaching and learning
- Applying concepts in music to other areas of life

Albert Einstein

- "If I were not a physicist, I would probably be a musician. I often think in music. I live my daydreams in music. I see my life in terms of music."

Others:

- "Music is the pleasure the human mind experiences from counting without being aware that it is counting." Leibniz
- "Besides language and music, mathematics is one of the primary manifestations of the free creative power of the human mind." Hermann Weyl

Pretest:

- What is frequency of an open A on a bass?
- Why causes instrument timbres to be different?
- What are log functions and how could they relate to music?
- How many possible combinations are there of a sequence of 4 notes? (ie: D,E,F,G)

Basic Categories

- Physical Properties of Music
  - Frequency, Amplitude, Duration, Timber
- The Perception of Music
  - How We Hear
- Music Composition
- The Materials of Music
  - Notes, Intervals, Scales
  - The Geometry of Playing Position

(From Musimathics, by Gareth Loy)

Physical Properties of Music

Acoustics and Energy

Energy (In many ways – indefinable)

- Energy in bowing
  - **energy** - (physics)The capacity of a physical system to do work.
  - Capacity of acting, operating, or producing an effect
  - *The great energies of nature are known to us only by their effects*
- *Another Bowing concept:*
  - *Force X Velocity=Power (or work/time)*
  - *Light bow – less velocity and less force*
  - *Heavy Bow – has more force, requires more velocity*

Kinetic Energy

- **Kinetic Energy** –the mechanical energy that a body has by virtue of its motion
  - Peak: Maximum Velocity (Zero Acceleration)
- **Potential Energy** – the energy that a body has due to its position in a force field
  - Peak: Maximum Acceleration (Zero Velocity)
- Spiccato and other off-string strokes

Gravity as it relates to bowing

- The state of having weight, heaviness
  - *(Physics)* The tendency of a mass of matter toward a center of attraction; esp. the tendency of a body toward the center of the earth.
  - Up bows vs. down bows

Activation Energy

- **Noun.** The energy that an atomic system must aquire before a process or reaction can occur; “catalysts” are said to reduce the energy of activation during the transition phase of a reaction.
- Many examples in the literature
- Holberg Suite, Grieg

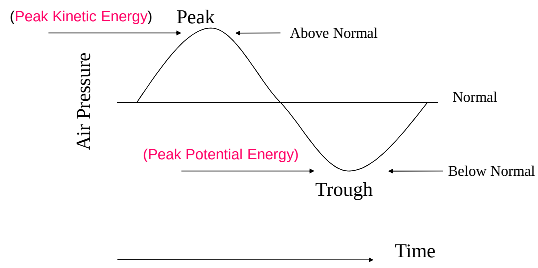
The Chorus Effect

- What happens when two people play instruments in unison? They are not always playing in precise synchronization, so there is some delay between the sounds they produce. In addition, the pitch of the two instruments can deviate somewhat, despite careful tuning.
- Electronic recreation of two musicians:
  - Y(n)=1/2 x(n)+ ½ x(n-d(n))
  - When n=variable delay time
  - 1 direct signal x(n)

Frequency

Pitch and mathematical relationships and values

Characteristics of Sound Waves



Characteristics of Sound Waves

- The physical distance from one peak to the next is a WAVELENGTH. Lower pitches have longer wavelengths (several feet) and higher pitches have shorter wavelengths (a few inches or less).

Characteristics of Sound Waves

- The number of cycles per second is the FREQUENCY.
- Frequency is measured in hertz (Hz) which is cycles per second.
- 1 thousand Hz = 1 kilohertz or 1 kHz
- Doubling the frequency raises the pitch by one octave
- The Height of the Wave is its AMPLITUDE (or volume)

Relating Frequency to Our Lives

- Low frequency tones have a low pitch. Low E on a bass is 41 Hz.
- High frequency tones have a high pitch. 4 octaves above middle C is 4186 Hz or 4.186 kHz.
- Children can hear frequencies from 20 Hz to 20 kHz. Adults can hear up to 15 kHz if they have good hearing.

A1 is 55 Hz

Find the 2nd, 3rd, 4th, and 5th octaves. (That is, the next 5 "A's" on the keyboard)

Relating Frequency to Our Lives

- The piano keyboard and frequency ranges:
  - The lowest A = 27.5 Hz
  - A1 = 55 Hz
  - A2 = 110 Hz
  - A3 = 220 Hz
  - A4 = 440 Hz
  - A5 = 880 Hz
  - A6 = 1760 Hz or 1.76 kHz
  - A7 = 3520 Hz or 3.52 kHz

Pythagoreans

- The Pythagoreans (ancient Greeks) discovered this:
- Multiplying a frequency by 3/2 raises a note by a Fifth
  - f=frequency, 2f=octave, 5th=3f/2
  - Therefore: 4th=$\frac{2f}{3/2f}$
- Also, Multiplying a note's frequency by 4/3 raises it by a Fourth.
- Major 3rd=5/4
- Major 6^{th}=5/3

Relating Frequency to Our Lives

- Middle C = 261.63 Hz
- Low E on a bass is 41Hz,Low E on a guitar is 82 Hz
- Electronic "Hum" is often at 60 Hz
- Look at the equalizer in your home or car stereo. How is it labeled?

A sine wave is a pure tone of a single frequency. It is usually generated by a tone generator. All sounds are *combinations* of different sine waves of different frequencies and amplitudes.

Complex Waves

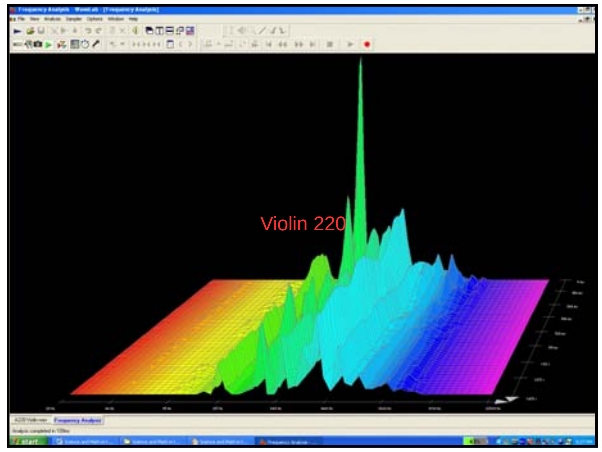
- The lowest frequency in a complex wave is called the FUNDAMENTAL.
- Higher frequencies in the complex wave are called OVERTONES or UPPER PARTIALS.
- Typically, the fundamentals have a much higher amplitude than the overtones

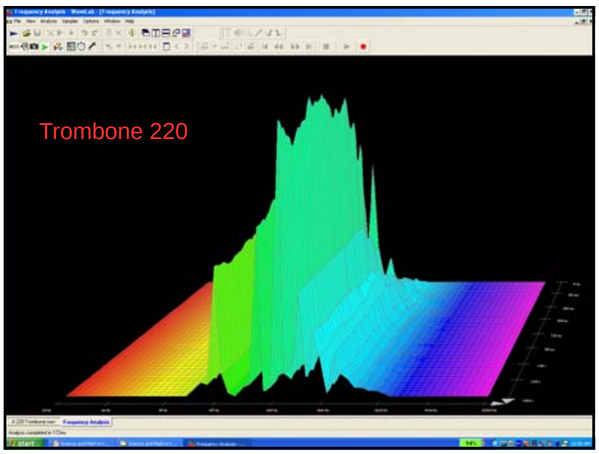
If the overtones are multiples of the fundamental frequency, they are called HARMONICS.

So, if the fundamental is 200Hz the 2^{nd} harmonic is 400Hz, the 3^{rd} is 600 Hz, etc.

Harmonics

- Why do harmonics make a difference?
- The harmonics and their amplitudes determine the TIMBRE of the sound of an instrument.
- This is what identifies a sound as a trumpet, violin, voice, etc.
- Noise has a wide band of frequencies and has an irregular, non-repeating wave form.



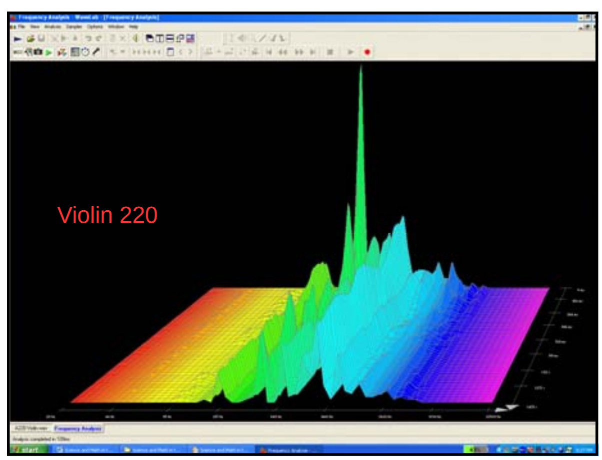


A4 is 220 Hz

Find the 2nd, 3rd, 4th, and 5th harmonics.

A4 is 220 Hz (Fundamental)

- 2ndHarmonic: 440 Hz
- 3rdHarmonic: 660 Hz
- 4thHarmonic: 880 Hz
- 5th Harmonic: 1100 Hz
- 6th Harmonic: 1320 Hz



A4 is 220 Hz (Fundamental)

- 2ndHarmonic: 440 Hz or A5
- 3rdHarmonic: 660 Hz or E5
- 4thHarmonic: 880 Hz or A6
- 5th Harmonic: 1100 Hz or C#6
- 6th Harmonic: 1320 Hz or E6

- The harmonics combine to form the basis of modern functional harmony

Harmonic relationships

- 2nd Harmonic is octave
- 3rd Harmonic is the "fifth" or the fifth note of the scale
- 5th Harmonic is the "third" or the third note of the scale
- The fundamental, third, and fifth combine to create a "major" chord
- Applications in arranging

Why do Octaves sound so similar?

- Answer: they share all of the same harmonics!
- A110 – 220, 330, 440, 550, 660, 770, 880
- A220 – 440, 880, 1760, etc

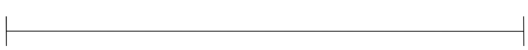
Tuning Systems

- Pythagorean tuning – 4ths and 5ths are very "in tune" Other intervals can suffer
- Just Tuning – More intervals have the ideal frequency intervals but some suffer greatly
- Equal Tempering – No interval other than octave will have the ideal frequency ratio
- The idea of "cents" was introduced in the 1800's by Alexander John Ellis

Harmonics and the String

A=220

(Fundamental and all harmonics)

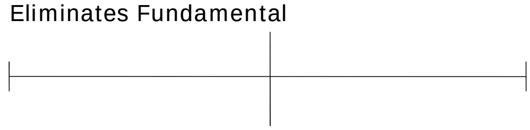


Harmonics and the String

A=220

String cut by ½ - A =440

Eliminates Fundamental

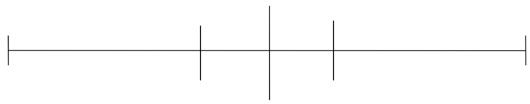


Harmonics and the String

A=220

String cut by 1/3 - E =660

Eliminates Fundamental and first harmonic



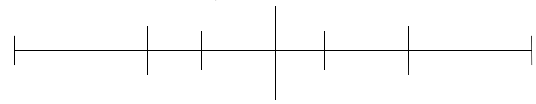
Harmonics and the String

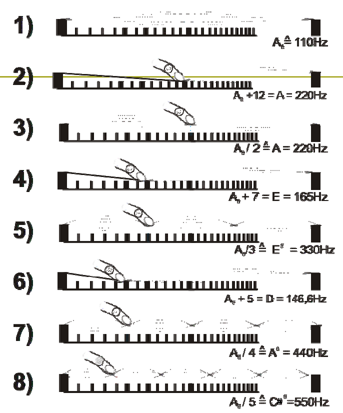
A=220

String cut by ½ - A =440

String cut by ¼ - A = 880 (on both sides)

Eliminates Fundamental, 1st and 2nd Harmonics



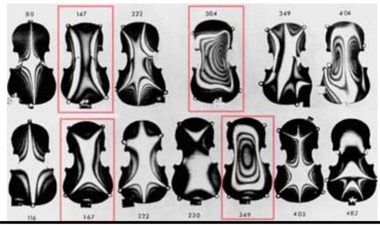


The Science of Violin Making

- Cremonia Revisited: The Science of Violin Making, Andrew Hsieh,
  - Engineering and Science, 2004, No. 4
  - Wonderful Article on the subject
    - Resonant Modes of a square plate
    - Types of Wood cuts (Quarter Cut, Slab Cut)
    - Scientific make-up of wood
    - Vibrational Modes
    - Frequency Analysis of 1725 Stradavari vs. 2002 Nagyvary
    - Effect of Varnish on the violin

Cremonia Revisited

- Vibrational Modes
  - X mode and Ring Modes are highlighted in red
  - Top Front Plate, Bottom is Back
  - Numbers are frequencies at which resonances occur
  - Sand placed on the instrument and collects at the nodes



Cremonia Revisited

- Frequency analysis
  - Showed that the high frequencies (harmonics) were much stronger between 1300Hz and 2500Hz in the Cremonese violins.
  - Another peak at the air resonance frequency
  - Demonstrated conclusively that there is a clear acoustical difference between Stadavari and Guarneri and contemporary violins.
  - This is a result of strong selective amplification of several key harmonics.
  - The cause of that amplification remains an open question!!

The Math of Finger Placement

- The historical technique is called the **rule of 18**, and it involves successively dividing the scale length minus the offset to the previous ½ step by **17.817**. This series of calculations puts the 12th ½ step (octave) at exactly half the scale length, and we know from the physics of vibrating strings that halving the vibrating string length of a theoretically perfect string doubles the frequency.
- d = s – (s / (2 ^ (n / 12))) where:

d = distance from nut;

s = scale length;

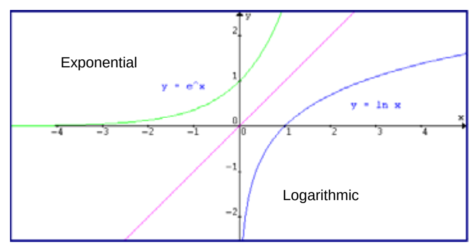
n = ½ step number; (1-12)

(Courtesy of NS Design Violins)

Perception of Music

Exponential and Log Functions

- What is an Exponential or Log Function???
- Let's Look at a Graph!

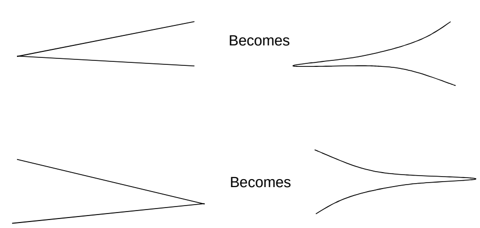


What Does this tell us?

- Early changes are very small
- Later changes are very large

- Where could this apply to music?

Crescendo and Diminuendo



The Decibel

Another Log function!

Decibel or dB

- Decibels are logarithmic in nature
- Small numbers are used to describe very large changes in power.
- Doubling the power (watts) results in an increase of 3 dB
- A 10 dB change is considered to be double apparent volume
- This requires 10 times the power!
- Decibel is 10log(P1/P2)

Music Composition

Math and Melody

Math and Melody

- Transpositions
  - Shifting Musical Patterns
  - Shorthand for "transposition by *n* semitones is T_n
  - So, a line that has been transposed down a 5th would be notated T_{-5}
  - Also T_{12} = T_0 Therefore, there are only 12 transpositions to deal with 0-11.
  - Think of the tune "Charge"
    - Tn, Tn+1, Tn+2, Tn+3, etc.
  - T_0 is called Identity Element

Math and Melody

- Retrograde "R"
  - Backwards –
    - ie Bach's Musical Offering
    - Beethoven Piano Sonata #29
    - Haydn Symphony 47 Minuet
  - Left to right flip
- Inversion "I"
    - Sousa Thunderer
    - Brahms Variations on a Theme of Robert Schumann
  - Up-down flip
  - More easily heard than retrogrades

Math and Melody

- The Math – putting all three together
  - $RR=T_0$
  - $T_nR=RT_n$
  - So, only 24 possible combinations of T's and R's exist. The twelve T's and the 12 T_nR's
  - Inversions, however, can be mirrored at many places
    - IT_n does not = T_nI
    - **Itn=T_{12-n}I**
    - So there are 48 possible T's, I's, and R's

Math and Melody

- The combinations of T's, I's and R's is called a GROUP
  - Many mathematical systems share these characteristics
    - The collection is closed with respect to combining its objects
    - The collection includes an Identity Element
    - Every object in the collection has an opposite
    - The combinations are *Associative*
      - *We can associate the middle term with either end term first and still get the same answer*

Math and Melody

- Ringing the Changes - Patterns

1x2x3…*xn*

# of Bells	# of Possible Combinations
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40,320

The Materials of Music

Basic Theory: Intervals

- Counting ½ steps -
  - Perfect Intervals:
    - 4th=5 ½ steps
      - +4th=6 ½ steps
      - Dim 4th=4 ½ steps
    - 5th=7 ½ steps
      - +5th=8 ½ steps
      - Dim 5th=6 ½ steps
    - Octave=12 ½ steps
      - +octave=13 ½ steps
      - Dim octave=11 ½ steps

Basic Theory: Intervals

- Counting ½ steps
  - Major 3rd=4 ½ steps
  - Minor 3rd=3 ½ steps
  - Dim 3rd=2 ½ steps
  - Aug 3rd=5 ½ steps
- Same for all Major/Minor Intervals
- P5 = 7 ½ steps
- So a M3 + m3 = P5

Intervals and Inversions

- What is the inversion of a 4th? C-F
  - 5th
- What is the inversion of a 2nd?
  - 7th
- What is the inversion of a 3rd?
  - 6th
- What is the pattern??

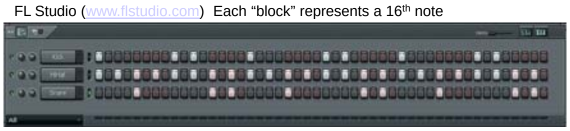
Time Signature and Sub-Division

Whole Note – One whole measure
Half Note – ½ of a whole note
Quarter Note – ¼ of a whole note

Whole (1)							
Half (½)				Half (½)			
Quarter ¼		Quarter ¼		Quarter ¼		Quarter ¼	
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

Time Signature – defines the basic division and emphasis of rhythmic beats in a piece of music

FL Studio (www.flstudio.com) Each "block" represents a 16th note



Duration

- Absolute duration
  - Quarter note = 60MM or beats per minute
  - Each quarter note lasts 1 second
  - Quarter note = 180 MM
    - How long does a quarter note last?
- Relative duration
  - Dotted Quarter = Quarter + Eighth
  - Dots:
    - In general, *n* dots after a note or rest of duration *D* indicate that the total duration *T* is:
    - $T=D(1/2^0+1/2^1+\ldots+1/2^n)$

The Geometry of Playing Position

- Violin
  - Perpendicular
  - Parallel
  - Rectangle
  - Pronate
- Cello
  - Equilateral
  - Parallel
- Others?

Further Topics
(from Musimathics Vol. II)

- Digital Audio and Sampling
- Complex Numbers
  - Representation of musical symbols
- Sound Synthesis
- Many, many others….

Post Test

- What is frequency of an open A on a bass?
  - 55Hz
- Why causes instrument timbres to be different?
  - Overtones or Harmonics
- What are log functions and how could they relate to music?
  - Small #'s that represent large changes
- How many possible combinations are there of a sequence of 4 notes? (ie: D,E,F,G)
  - 4*3*2*1=24

Special thanks to:

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